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# Non-local charges on $AdS_5 \times S^5$ and PP-waves

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Abstract: We show the existence of an infinite set of non-local classically conserved charges on the Green-Schwarz closed superstring in a pp-wave background. We find that these charges agree with the Penrose limit of non-local classically conserved charges recently found for the $AdS_5 \times S^5$ Green-Schwarz superstring. The charges constructed in this paper could help to understand the role played by these on the full $AdS_5 \times S^5$ background.

Keywords: Conformal Field Models in String Theory, Penrose limit and pp-wave background, AdS-CFT and dS-CFT Correspondence, Global Symmetries.

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## 1. Introduction

The AdS/CFT correspondence [1] expresses the equivalence between type-IIB string theory compactified in the background $AdS_5 \times S^5$ and pure $\mathcal{N} = 4$ SYM gauge theory in four dimensions. In order to gain a better understanding of such duality, it is of interest to exactly solve either side of the correspondence.

In the last year, clues for the existence of integrable structures on both sides of the duality have been pointed out. For instance, hints of the integrability of $\mathcal{N} = 4$ SYM in the large-$N$ limit were given by studying the dilatation operator in perturbation theory[1] and it was determined that the one loop mixing matrix for anomalous dimensions can be identified with the hamiltonian of an integrable spin chain [3].

On the other hand, from the string theory side, after some clue from the bosonic theory [5] , Bena *et. al.* found an infinite set of non-local classically conserved charges for the Green-Schwarz superstring on $AdS_5 \times S^5$ [4].[2] This would imply that the world-sheet theory is a integrable system.

To understand the structure of such charges, their gauge theory dual, etc, and, eventually, to use them in order to solve string theory on $AdS_5 \times S^5$ is perhaps a very difficult

[1]In fact the integrability of one-loop multi-particle renormalization was discovered first in QCD five years ago in a serie of papers [2].

[2]This kind of charges exists also for the pure spinor superstring [6, 7].

task. A warming up exercise would be to study the much simpler problem of such charges on the pp-wave background. This background is simple enough to be an exact solution of string theory, so one should be able to understand these charges, their dual, how to use them to solve the theory, etc, in this limit. On the other hand, the pp-wave background is rich enough to admit infinite number of classical conserved non-local charges and, apparently, to provide us with some non trivial information about the role of these in the $AdS_5 \times S^5$ background.

The aim of this paper is to study the structure of these charges and give an algorithm that allows us to write the explicit form of these at arbitrary order. We do it in two different ways: first, by constructing a set of non-local charges from the world-sheet sigma model of closed string theory on pp-waves backgrounds.[3] Since we need to impose periodic boundary conditions we have to take the trace (or some invariant cyclic operator) of the product of generators, of the Lie algebra, appearing in the charge. As the algebra of the pp-wave is non semi simple, by just taking the trace as the invariant we obtain trivial charges, since the trace is degenerate. For this algebra we have, however, a non degenerate form, that can be used in order to obtain non trivial conserved charges. The second method consists of constructing the charges on $AdS_5 \times S^5$ and then taking their Penrose limit. In this case we can take the trace as the invariant, since the algebra is semi-simple. We show, up to the first non trivial order, that both results agree.

When considering an uncompactified sigma model the first non-local charges generate (under repeated Poisson-Dirac brackets) an infinite dimensional algebra, called the Yangian, and so we generate in this way the complete tower of non-local charges [15]. Such algebra appears also when considering operators acting on spin chains. A relation between both approaches was found in [16]. When one considers periodic boundary conditions the structure of the charges changes, and it is not clear whether one can generate the complete tower from a finite number of charges. Indeed, for the first order charges this is not possible.

This paper is organized as follows, in the next section we give a brief introduction to the construction of non-local classical conserved charges for the Green-Schwarz superstring on $AdS_5 \times S^5$ and argue that such charges should also exist for the pp-wave. In section 3 we show how write the explicit form of these charges in the light cone gauge and we do it for the first non trivial orders. In section 4 we write the explicit form for the first non trivial charges for $AdS_5 \times S^5$ and show that its Penrose limit coincides with the charge previously found. We also check explicitly that the semiclassical value for that AdS charge for a rotating string on $S^5$ (the dual of a BMN state) coincides with that of the pp-wave charge when applied to the same BMN state. We conclude with some discussion and open problems.

## 2. Nonlocal charges in coset models

In this section we review the construction of non-local charges for the Green-Schwarz superstring on $AdS_5 \times S^5$. We recall the results found in [4] that will be useful in the

[3]Somehow related, the symmetries of D-branes on pp-waves backgrounds was studied in [8] where it was determined that the open string admits an infinite number of non-local symmetries.



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following and then we show that the Green-Schwarz superstring on pp-waves possesses charges of the same structure.

### 2.1 Nonlocal charges in $AdS_5 \times S^5$

The Green-Schwarz superstring on $AdS_5 \times S^5$ can be considered as a non-linear sigma model where the field $g(x)$ takes values in the coset superspace [9, 10, 11]:

$$\frac{G_{AdS}}{H_{AdS}} = \frac{\mathrm{PSU}(2,2|4)}{\mathrm{SO}(4,1)\times \mathrm{SO}(5)}\,, \tag{2.1}$$

whose bosonic part is

$$\frac{\mathrm{SO}(4,2)}{\mathrm{SO}(4,1)} \times \frac{\mathrm{SO}(6)}{\mathrm{SO}(5)} = AdS_5 \times S^5\,. \tag{2.2}$$

The bosonic generators of $G_{AdS}$ are the translations $P^a$ and rotations $J^{ab}$, with $a, b = 0, \ldots, 4$, (generators of SO(4,2)) and translations $P^{a'}$ and rotations $J^{a'b'}$, with $a', b' = 0, \ldots, 4$, (generators of SO(6)). The fermionic generators are 32 spinors $Q^{\alpha\alpha',I}$ with $\alpha, \alpha' = 1, \ldots, 4$ and $I = 1, 2$. $H_{AdS}$ is the stability subgroup of $G_{AdS}$, generated by the rotations $J^{ab}$ and $J^{a'b'}$.

Then, it follows that the Lie algebra of PSU(2,2|4) can be decomposed in the following way:

$$\mathcal{G}_{AdS} = \mathcal{H}_{AdS} + \mathcal{P} + \mathcal{Q}_1 + \mathcal{Q}_2\,, \tag{2.3}$$

with $\mathcal{H}_{AdS}$ the Lie algebra of $H_{AdS}$, $\mathcal{P}$ the algebra of the translations, and $\mathcal{Q}_1$ and $\mathcal{Q}_2$ two copies of the $(4,4)$ of $\mathcal{H}_{AdS}$.

We focus on the current

$$J = -g^{-1}\partial g = H + P + Q_1 + Q_2\,. \tag{2.4}$$

Using the relation $dJ = J \wedge J$ and the $\mathcal{Z}_4$ grading respected by the algebra,[4] we can find equations for $dH$, $dP$, $dQ_1$ and $dQ_2$. In terms of the lower-case currents, defined as $x = gXg^{-1}$ they read:[5]

$$\begin{aligned}
dh &= -h\wedge h + p\wedge p - h\wedge p - p\wedge h - h\wedge q - q\wedge h + \frac{1}{2}(q\wedge q - q'\wedge q')\,,\\
dp &= -2p\wedge p - p\wedge q - q\wedge p + \frac{1}{2}(q\wedge q + q'\wedge q')\,,\\
dq &= -2q\wedge q\,,\\
dq' &= -2p\wedge q' - 2q'\wedge p - q\wedge q' - q'\wedge q\,,
\end{aligned} \tag{2.5}$$

where we have defined $q = q_1 + q_2$ and $q' = q_1 - q_2$. This can be supplemented with the equations of motion [12]

$$d{*}p = p\wedge {*}q + {*}q\wedge p + \frac{1}{2}(q\wedge q' + q'\wedge q)\,,$$

[4] Under such grading $\mathcal{H}$, $\mathcal{P}$, $\mathcal{Q}_1$ and $\mathcal{Q}_2$ have charge 0, 2, 1 and 3 respectively.

[5] Capital letters, X, denote currents that are conjugated by right multiplication, and they in general correspond to some decomposition under representations of the Lie Algebra, lower case letters, x, correspond to currents conjugated by left multiplication and in general do not have a simple decomposition under representations.



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$$
\begin{aligned}
0 &= p \wedge (*q - q') + (*q - q') \wedge p \,, \\
0 &= p \wedge (q - *q') + (q - *q') \wedge p \,.
\end{aligned} \tag{2.6}
$$

Next, we define

$$
a = \alpha p + \beta * p + \gamma q + \delta q' \,, \tag{2.7}
$$

then, by requiring $a$ to be a flat connection, i.e. $da + a \wedge a = 0$, we find two one-parameter families of solutions, given by

$$
\begin{aligned}
\alpha &= -2\sinh^2\lambda \,, \\
\beta &= \mp 2\sinh\lambda\cosh\lambda \,, \\
\gamma &= 1 \pm \cosh\lambda \,, \\
\delta &= \sinh\lambda \,.
\end{aligned} \tag{2.8}
$$

Given a flat connection, the following equation

$$
dU = -aU \,, \tag{2.9}
$$

is integrable, on a simply connected space, and with initial condition $U(x_0, x_0) = 1$, then

$$
U(x, x_0) = \mathcal{P}\exp\left(-\int_C a\right) . \tag{2.10}
$$

With $C$ a path from $x_0$ to $x$ and $\mathcal{P}$ the path ordering in the Lie algebra. This allows the construction of an infinite number of conserved charges, given by

$$
Q^{a\pm}(t) = U^{a\pm}(\infty, t; -\infty, t) \,. \tag{2.11}
$$

This charges can be shown to be conserved, for an appropriate falloff of the fields at infinity.

As we are interested in closed string theory, the world-sheet respects periodic boundary conditions. Considering the product of four Wilson lines that form a closed path in the strip (and don't enclose any singularity) we have

$$
U^{a\pm}(0, t_0; 0, t)U^{a\pm}(0, t; 1, t)U^{a\pm}(1, t; 1, t_0) = U^{a\pm}(0, t_0; 1, t_0) \,, \tag{2.12}
$$

from this equation it is easy to see that the following quantity

$$
Q^{a\pm}(t) = \langle U^{a\pm}(0, t; 1, t)\rangle \,, \tag{2.13}
$$

is conserved. Here the spatial coordinate $\sigma$ is restricted to the $[0, 1]$ interval and we have assumed periodic boundary conditions. With $\langle\mathcal{O}\rangle$ we denote some invariant cyclic operator. For the case of $AdS_5 \times S_5$, whose algebra of isometries is a semi-simple algebra, the operator $\langle\ \rangle$ can simply be the trace.

A convenient way to see the infinite set of charges is by Taylor expanding in the parameter $\lambda$, for instance

$$
Q^{a-}(t) = 1 + \sum_{n=1} \lambda^n Q_n \,. \tag{2.14}
$$

Writing $a = \lambda a^{(1)} + \lambda^2 a^{(2)} + \cdots$, for the first order charges we have

$$Q_1 = \int_0^1 d\sigma a_1^{(1)}(\sigma)\,, \tag{2.15}$$

$$Q_2 = \int_0^1 d\sigma a_1^{(2)}(\sigma) + \int_0^1 d\sigma \int_0^\sigma d\sigma' a_1^{(1)}(\sigma) a_1^{(1)}(\sigma')\,, \tag{2.16}$$

and so on.

### 2.2 Nonlocal charges on the PP-wave

In this subsection we will argue that these non-local charges exist also for the case of the pp-wave and have the same form (2.13).

Let us consider the Green-Schwarz superstring on a pp-wave RR background

$$ds^2 = 2dx^+dx^- - m^2 x_I^2 dx^{+2} + dx_I^2 \tag{2.17}$$

$$F^{-i_1\dots i_4} = 2m\epsilon^{i_1\dots i_4}\,, \qquad F^{-i'_1\dots i'_4} = 2m\epsilon^{i'_1\dots i'_4} \tag{2.18}$$

$I = 1,\dots 8$, describes the eight flat directions of the pp-wave, $i,j = 1,\dots 4$ and $i',j' = 5,\dots,8$. Here $m$ is a dimensionful parameter, that from now on we will take equal to one, and $x^+$ is taken to be the light-cone evolution parameter.

The Green-Schwarz superstring on a pp-wave background can be regarded as a non linear sigma model on the coset superspace [12]

$$\frac{G}{H}\,. \tag{2.19}$$

The transformation group $G$ is spanned by the following generators: The even (bosonic) part of the superalgebra includes ten translation generators $P^\mu$, SO(4) rotation generators $J^{ij}$, $i,j = 1,\dots,4$, $SO'(4)$ rotation generators $J^{i'j'}$, $i',j' = 5,\dots,8$ and eight rotation generators in the $(x^-, x^I)$ plane $J^{+I}$. The odd (fermionic) part of the superalgebra consists of the complex 16-component spinor $Q_\alpha$, $\alpha = 1,\dots 16$. The stability group $H$, is generated by $J^{ij}$, $J^{i'j'}$ and $J^{+I}$. The algebra between the relevant generators is given in the appendix.

Then, it follows that the Lie algebra $\mathcal{G}$ can be decomposed as follows

$$\mathcal{G} = \mathcal{H} + \mathcal{P} + \mathcal{Q}_1 + \mathcal{Q}_2\,. \tag{2.20}$$

Hence the current $J$ can be decomposed as in (2.4). The algebra in this case also respects a $\mathcal{Z}_4$ grading that together with the condition $dJ = J \wedge J$ leads to equations of the exact form of (2.5).

For simplicity, from now on we will deal explicitly with the bosonic fields, however the discussion can be carried out for the fermionic fields as well. The equation of motion reads [12]:

$$d * p = 0\,, \tag{2.21}$$

which is the same as (2.6). We can then construct the same set of nonlocal conserved charges (2.13), from the connection

$$a = \alpha p + \beta * p\,, \tag{2.22}$$

with $\alpha$ and $\beta$ given by (2.8). In terms of Cartan 1-forms

$$p = L^\mu p_\mu, \qquad p_\mu = gP_\mu g^{-1}\,, \tag{2.23}$$

with $L^\mu$ 1-forms in the two dimensional world-sheet and $P_\mu$ generators of $G$. We know that string theory on this background is exactly solvable in the light cone gauge, so it is interesting to ask what is the form of these charges when we fix such gauge.

## 3. Explicit form of the charges on the PP-wave

In the light cone gauge, the Cartan 1-forms become

$$L^+ = dx^+, \qquad L^I = dx^I\,, \qquad L^- = dx^- - \frac{1}{2}x_I^2 dx^+\,, \tag{3.1}$$

with $x^\mu$ two dimensional fields on the world-sheet, depending on general on the world-sheet coordinates $(\tau,\sigma)$ with metric $g_{ab}$. Further we fix

$$\sqrt{g}g^{ab} = \eta^{ab}, \qquad x^+(\tau,\sigma) = \tau, \qquad p^+ = 1\,. \tag{3.2}$$

With this, the flat connection takes the form

$$a = \alpha p + \beta(*p) = \pm 2\lambda(*p) - 2\lambda^2 p \pm \frac{4}{3}\lambda^3 + \cdots\,, \tag{3.3}$$

with

$$p = d\tau p_+ + dx^I p_I + \left(dx^- - \frac{1}{2}x_I^2 d\tau\right)p_-\,. \tag{3.4}$$

It is important to notice that $p$ is written in terms of lower-case generators, of the form $p^\mu = gP^\mu g^{-1}$, and hence dependent on the world-sheet coordinates via $g$. Choosing a coset representative of the form $g(x^\mu) = e^{\tau P^-}e^{x^I P^I}$ we can express the lower-case generators in terms of upper-case generators as follows

$$p^I = \cos\tau P^I - \sin\tau J^{+I}\,, \tag{3.5}$$

$$p^- = P^- - \frac{x^I x^I}{2}P^+ + x^I\cos\tau J^{+I} + x^I\sin\tau P^I\,, \tag{3.6}$$

$$p^+ = P^+\,. \tag{3.7}$$

Writing these charges in terms of upper-case generators, we see their explicit dependence on the world-sheet coordinates, then by Taylor expanding in the parameter $\lambda$ we can construct the charges order by order.

### 3.1 First order charge

For the first order in $\lambda$ we obtain the charge

$$Q_1 = \left\langle \int_0^1 d\sigma(P^- + A(\sigma)P^+ + B^I(\sigma)P^I + C^I(\sigma)J^{+I}) \right\rangle, \tag{3.8}$$

with

$$A(\sigma) = \partial_\tau x^- - x^I x^I \tag{3.9}$$

$$B^I(\sigma) = \sin\tau x^I + \cos\tau\partial_\tau x^I \tag{3.10}$$

$$C^I(\sigma) = \cos\tau x^I - \sin\tau\partial_\tau x^I \tag{3.11}$$

where we are not showing the $\sigma$ dependence of the world-sheet fields. By using (A.13) we can write $A(\sigma)$ purely in terms of the fields $x^I$

$$A(\sigma) = -\frac{1}{2}(\partial_\tau x^I\partial_\tau x^I + \partial_\sigma x^I\partial_\sigma x^I + x^I x^I)\,. \tag{3.12}$$

By using the equations of motion, we can see that the coefficient of every generator is a classically conserved charge. So we have four conserved quantities

$$Q_1 = 1\,, \qquad Q_A = \int_0^1 d\sigma A(\sigma)\,, \qquad Q_{B^I} = \int_0^1 d\sigma B^I(\sigma)\,, \qquad Q_{C^I} = \int_0^1 d\sigma C^I(\sigma)\,. \tag{3.13}$$

Since every coefficient is conserved, this implies the conservation of $Q_1$, for every invariant $\langle\ \rangle$ we choose.

It is interesting to notice that if we plug the mode expansion of the fields $x^I$ (see appendix) we find the following expressions for the charges

$$Q_A = \frac{1}{2}\left(p_0^I p_0^I + x_0^I x_0^I\right) + \sum_{n\neq 0}\left(\alpha_n^{I1}\alpha_{-n}^{I1} + \alpha_n^{I2}\alpha_{-n}^{I2}\right), \tag{3.14}$$

$$Q_{B^I} = p_0^I\,, \qquad Q_{C^I} = x_0^I\,. \tag{3.15}$$

The classical Poisson-Dirac brackets among these charges are given by

$$[Q_{B^I}, Q_{C^J}] = \delta^{IJ} Q_T\,, \qquad [Q_{B^I}, Q_A] = Q_{C^I}\,, \qquad [Q_A, Q_{C^I}] = Q_{B^I}\,, \tag{3.16}$$

whereas, of course, $Q_T$ has zero Poisson-Dirac bracket with every operator. Note that this is the same algebra followed by the bosonic generators of the pp-wave algebra.

From (3.14) we see that these three charges represent the constants of motion $p_0^I$, $x_0^I$ and the hamiltonian, that are, of course, the quantities associated with the symmetries of the pp-wave.

### 3.2 Second order charge

For the second order charge we have[6]

$$Q_2 = \left\langle \int_0^1 d\sigma \int_0^\sigma d\sigma' (P^- + A(\sigma)P^+ + B^I(\sigma)P^I + C^I(\sigma)J^{+I}) \times \right.$$
$$\left. \times\,(P^- + A(\sigma')P^+ + B^J(\sigma')P^J + C^J(\sigma')J^{+J}) \right\rangle.$$

[6] $Q_2$ has in general a single integral term, proportional to $\int p_1$, however, this is 0 in this case, since $p_1$ is a total derivative in the spatial coordinate.



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In order to study $Q_2$ let us introduce the following notation:

$$Q_{\mathcal{A}\mathcal{B}} = \int_0^1 d\sigma \int_0^\sigma d\sigma' \mathcal{A}(\sigma)\mathcal{B}(\sigma')\,, \tag{3.17}$$

where $\mathcal{A}, \mathcal{B}$ can take the values $1, A, B^I$ or $C^I$. Then we find the following conserved quantities

$$\begin{gathered}
Q_{11}\,, \qquad Q_{AA}\,, \qquad Q_{BB}\,, \qquad Q_{CC} \\
Q_{\{1,A\}} = Q_{1A} + Q_{A1}\,, \qquad Q_{\{1,B\}} = Q_{1B} + Q_{B1}\,, \qquad Q_{\{1,C\}} = Q_{1C} + Q_{C1}\,, \\
Q_{\{A,B\}} = Q_{BA} + Q_{AB}\,, \qquad Q_{\{A,C\}} = Q_{AC} + Q_{CA}\,, \qquad Q_{\{B,C\}} = Q_{BC} + Q_{CB}\,, \\
Q_a = \frac{1}{2} Q_{[C^I,B^I]} \left\langle P^+ \right\rangle + Q_{[B^I,1]} \left\langle J^{+I} \right\rangle + Q_{[1,C^I]} \left\langle P^I \right\rangle .
\end{gathered}$$

where we have introduced the notation $Q_{[\mathcal{A},\mathcal{B}]} = Q_{\mathcal{A}\mathcal{B}} - Q_{\mathcal{B}\mathcal{A}}$. The charges in the first three lines, can be written as product of the charges appearing in $Q_1$, in fact

$$\begin{aligned}
Q_{\{\mathcal{A},\mathcal{B}\}} &= \int_0^1 d\sigma \int_0^\sigma d\sigma' \mathcal{A}(\sigma)\mathcal{B}(\sigma') + \int_0^1 d\sigma \int_0^\sigma d\sigma' \mathcal{B}(\sigma)\mathcal{A}(\sigma') \\
&= \int_0^1 d\sigma \int_0^1 d\sigma' \mathcal{A}(\sigma)\mathcal{B}(\sigma') = Q_{\mathcal{A}} Q_{\mathcal{B}}\,,
\end{aligned} \tag{3.18}$$

where we have used that $\mathcal{A}$ and $\mathcal{B}$ commute. As for the last charge, note that since the invariant operator in cyclic, then $\langle T^A T^B \rangle - \langle T^B T^A \rangle = \langle [T^A, T^B] \rangle = 0$ and it becomes trivial.

In order to get some non-trivial conserved charge we should go to higher order.

### 3.3 Third and higher order charges

In this subsection we try to determine the general structure of the higher order charges and give an algorithm to write them explicitly.

As seen in the previous section, the charge of order $N$ will have a local contribution (only one integral) plus a bi-local (two integrals), etc, up to a N-local contribution, given schematically by sum of terms of the form

$$Q_{A_1 \ldots A_N} = \int_0^1 d\sigma_1 \mathcal{A}_1 \int_0^{\sigma_1} d\sigma_2 \mathcal{A}_2 \cdots \int_0^{\sigma_{N-1}} d\sigma_N \mathcal{A}_N\,, \tag{3.19}$$

times the corresponding product of generators, plus all the permutations.

In order to study these combinations, let us notice that we can write an arbitrary permutation in the following form

$$\begin{aligned}
Q_{A_{I_1} \ldots A_{I_N}} &= \int_0^1 d\sigma_1 \mathcal{A}_1 \int_{\sigma_1^i}^{\sigma_1^{i+1}} d\sigma_2 \mathcal{A}_2 \cdots \int_{\sigma_{N-1}^j}^{\sigma_{N-1}^{j+1}} d\sigma_N \mathcal{A}_N \qquad (3.20) \\
Q_{A_{I_1} \ldots A_{I_p} B A_{I_{p+1}}, \ldots A_{I_N}} &= \int_0^1 d\sigma_1 \mathcal{A}_1 \int_{\sigma_1^i}^{\sigma_1^{i+1}} d\sigma_2 \mathcal{A}_2 \cdots \int_{\sigma_{N-1}^j}^{\sigma_{N-1}^{j+1}} d\sigma_N \mathcal{A}_N \int_{\sigma_N^{N-p}}^{\sigma_N^{N-p+1}} d\sigma_{N+1} \mathcal{B}\,,
\end{aligned}$$

with $\sigma_m^i$ taking the values $0, \sigma_1, \ldots, \sigma_m, 1$ in crescent order. In other words, we express the permutations by interchanging the intervals of integration instead of the order of the $\mathcal{A}_i$.

We can give a precise recursive relation giving the integral of any permutation if we complement (3.20) with the following relation

$$\int_0^1 d\sigma \mathcal{B}(\sigma) \int_0^\sigma d\sigma_1 \mathcal{A}(\sigma_1) = \int_0^1 d\sigma \mathcal{A}(\sigma) \int_\sigma^1 d\sigma_1 \mathcal{B}(\sigma_1) \,. \tag{3.21}$$

For such a N-local integral, we have $N!$ permutations, however, not all of them are independent of integrals appearing at lower order. For instance, with the recursive relation given here, it is easy to see that the completely symmetric sum of all the permutations is just the product of the local integrals

$$Q_{A_1 \ldots A_N} + \text{permutations} = Q_{A_1}, \ldots, Q_{A_N} \,. \tag{3.22}$$

More generally, one can prove that

$$Q_{A_1 \ldots A_N B} + Q_{A_1 \ldots A_{N_1} B A_N} + \cdots + Q_{B A_1 \ldots A_N} = Q_B Q_{A_1 \ldots A_N} \,, \tag{3.23}$$

from where (3.22) as well as other relations can be shown. By using (3.23) together with the commutation relation among the generators one can write an arbitrary order charge in a way in which lower order contributions are explicit. For instance, for the third order charge we have[7]

$$\begin{aligned} Q_3 &= Q_{ABC} \left\langle P^A P^B P^C \right\rangle \\ &= \frac{1}{6} \left( Q_{ABC} \left\langle P^A P^B P^C \right\rangle + \text{permutations} \right) \\ &= \frac{1}{12} Q_A Q_B Q_C \left\langle P^A \{P^B, P^C\} \right\rangle + \frac{1}{12} Q_A Q_{[B,C]} \left\langle P^A [P^B, P^C] \right\rangle + \frac{1}{6} Q_{A[B,C]} \left\langle P^A [P^B, P^C] \right\rangle \\ &= \frac{1}{12} Q_A Q_B Q_C \left\langle P^A \{P^B, P^C\} \right\rangle + \frac{1}{4} Q_A Q_{[B,C]} \left\langle P^A [P^B, P^C] \right\rangle . \end{aligned} \tag{3.24}$$

As happened for $Q_2$, the first contribution of the right-hand side of (3.24) is conserved, independently of the choice of $\langle \ \rangle$, since it is the product of conserved charges. Let us focus in the nontrivial piece

$$Q_3^{NT} = Q_A Q_{[B,C]} \left\langle P^A, [P^B, P^C] \right\rangle = Q_A Q_{[B,C]} f_D^{BC} \left\langle P^A P^D \right\rangle . \tag{3.25}$$

At this point we need to give an expression for $\left\langle P^A P^D \right\rangle$, that we will call $\Omega^{AD}$. In our case $P^A$ can take the values $P^I, J^{+I}, P^-$ and $P^+$. If we take $\Omega^{AB} = Tr(P^A P^B)$ , then we obtain

$$\Omega^{AB} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

[7] There will be also a local term, that is proportional to $Q_1$, and a bi-local term, whose contribution vanish for the case under consideration.

and as can be easily seen, we obtain trivial conserved charges. The fact that $\Omega^{AB}$ is degenerate is due to the fact the pp-wave algebra is non semi-simple. Fortunately, the most general $\Omega^{AB}$ with the required properties has been given for the algebra under consideration [13, 14][8]

$$\Omega^{AB} = \begin{pmatrix} k & 0 & 0 & 0 \\ 0 & k & 0 & 0 \\ 0 & 0 & b' & k \\ 0 & 0 & k & 0 \end{pmatrix}$$

With this choice one can see that $Q_3^{NT}$ is non trivial and conserved:

$$\begin{aligned} Q_3^{NT} &= Q_{[C^I,B^I]}Q_1 + Q_{[B^I,1]}Q_{C^I} + Q_{[1,C^I]}Q_{B^I} \\ &= -2\int_0^1 d\sigma \int_0^1 d\sigma' \sigma x^I(\sigma)\partial_\tau x^I(\sigma') + 2\int_0^1 d\sigma \int_0^1 d\sigma' \sigma \partial_\tau x^I(\sigma) x^I(\sigma') + \\ &\quad + \left(\int_0^1 d\sigma \int_0^\sigma d\sigma' x^I(\sigma)\partial_\tau x^I(\sigma') - \int_0^1 d\sigma \int_0^\sigma d\sigma' \partial_\tau x^I(\sigma) x^I(\sigma')\right). \end{aligned} \tag{3.26}$$

Written in terms of the mode expansion

$$Q_3^{NT} = \sum_{n\neq 0} \frac{2}{w_n k_n}\left(\alpha_n^{I1}\alpha_{-n}^{I1} - \alpha_n^{I2}\alpha_{-n}^{I2}\right). \tag{3.27}$$

In order to evaluate higher order charges, one should give an expression for higher order invariants. In general they will have contribution from lower order invariants, plus some independent piece. We stress that in general, for a given order charge, there will be terms conserved by themselves, for instance, at fourth order we find

$$Q_4^I = \int_0^1 d\sigma x^I(\sigma) \int_0^\sigma d\sigma' x^I(\sigma') + \int_0^1 d\sigma \partial_\tau x^I(\sigma) \int_0^\sigma d\sigma' \partial_\tau x^I(\sigma'), \tag{3.28}$$

plus some other complicated contributions. Note that even if the complete charges of a given order is a Casimir of the group, there will be components that are conserved by themselves and need not to be a Casimir. Plugging the oscillator expressions for the fields $x^I$ we obtain

$$Q_4^I = \frac{1}{2}(x_0^I x_0^I + p_0^I p_0^I) - i\sum_m \frac{k_n}{w_n^2}(\alpha_n^1\alpha_{-n}^1 - \alpha_n^2\alpha_{-n}^2). \tag{3.29}$$

From the oscillator expressions for $Q_3^{NT}$ and $Q_4^I$ it is evident that they will have vanishing classical Poisson Dirac brackets between them, so we see that they will not generate new charges. This is different to the situation of sigma models with boundary conditions at infinity, where all the infinite set of classically conserved non-local charges is generated by the first non-local charges.

As we will see in the next section, there is another procedure to recover such charges, that simply consist in taking the Penrose limit of the corresponding charges on $AdS_5 \times S^5$. In this case the algebra is semi-simple, and so we can take the invariant operator as the trace.

[8] For the product of $P^IP^J$, or $J^{+I}J^{+J}$, we take the invariant to be proportional to $\delta^{IJ}$.

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## 4. $AdS_5 \times S^5$ charges and their Penrose limit

In the previous section we have given the explicit form for the infinite set of non-local classically conserved charges for the pp-wave by studying directly string theory on such background. It is expected that such charges are the Penrose limit of the charges for the $AdS_5 \times S^5$. In this section we will prove that this is the case for the first non-local charge.

### 4.1 Explicit form of the charges on $AdS_5 \times S_5$

As before, the first order charge can be written as (we are forgetting about the trace)

$$Q^1_{AdS} = \int (L_a p^a + L_{a'} p^{a'}) \,, \tag{4.1}$$

with the Cartan 1-forms given by [17]

$$L^a_{\rm AdS} = dy^a + \left(\frac{\sinh y}{y} - 1\right) dy^b \boldsymbol{\Upsilon}_b{}^a \,, \qquad L^{a'}_{\rm AdS} = dy^{a'} + \left(\frac{\sin y'}{y'} - 1\right) dy^{b'} \boldsymbol{\Upsilon}_{b'}{}^{a'} \,, \tag{4.2}$$

with

$$\begin{aligned} y &= \sqrt{y^2} = \sqrt{y^a y_a}\,, & y' &= \sqrt{y'^2} = \sqrt{y^{a'} y_{a'}}\,, \\ \boldsymbol{\Upsilon}^b_a &= \delta_a{}^b - \frac{y_a y^b}{y^2}\,, & \boldsymbol{\Upsilon}^{b'}_{a'} &= \delta_{a'}{}^{b'} - \frac{y_{a'} y^{b'}}{y'^2}\,. \end{aligned} \tag{4.3}$$

In this coordinates the bosonic lagrangian (equivalently the metric) reads:

$$\mathcal{L} = (dy)^2 + (\sinh y)^2 d\Omega_4^2 + (dy')^2 + (\sin y')^2 d'\Omega^2_{4'} \tag{4.4}$$

where the 4-sphere metrics are given by

$$d\Omega_4^2 = \frac{(dy^a)(dy^a) - (dy)^2}{y^2}\,, \tag{4.5}$$

and the same for $d'\Omega^2_{4'}$. As before, in order to show the explicit dependence on the world-sheet coordinates it is convenient to express lower-case generators in terms of upper-case generators, for the present case we obtain

$$p^a = \cosh y P^a + \left(\frac{1-\cosh y}{y^2}\right) y^a y^b P^b - \frac{\sinh y}{y} y^b J^{ab}\,, \tag{4.6}$$

$$p^{a'} = \cos y' P^{a'} + \left(\frac{1-\cos y'}{y'^2}\right) y^{a'} y^{b'} P^{b'} + \frac{\sin y'}{y'} y^{b'} J^{a'b'}\,. \tag{4.7}$$

Now in terms of upper-case generators

$$Q^1_{AdS} = \int (C_a P^a + C_{ab} J^{ab} + C'_a P^{a'} + C_{a'b'} J^{a'b'})\,, \tag{4.8}$$

with

$$C_a = \cosh y L_a + \left(\frac{1-\cosh y}{y^2}\right) y^b dy^b y^a\,, \qquad C_{ab} = -\frac{\sinh y}{y}\left(L^a y^b - L^b y^a\right)\,, \tag{4.9}$$

$$C_{a'} = \cos y' L_{a'} + \left(\frac{1-\cos y'}{y'^2}\right) y^{b'} dy^{b'} y^{a'}\,, \qquad C_{a'b'} = \frac{\sin y'}{y'}\left(L^{a'} y^{b'} - L^{b'} y^{a'}\right). \quad (4.10)$$

So the charges $\int_0^1 d\sigma C_{a,0}$, etc, are conserved quantities, and they represent the isometries of theory. As it is well know, by performing the Penrose limit, the isometries of $AdS_5 \times S_5$ map into the isometries of the pp-waves, so the Penrose limit of this first order charges are the first order charges found previously.

In order to find higher order charges we should worry about the invariant $\langle\ \rangle$. The algebra of $AdS_5 \times S_5$ is semi-simple, so we can simply take the trace of product of upper-case operators. Again the second order charge will be trivial and we should go to the third order.

Let us write the upper-case generators as $T^a$, then $[T^a, T^b] = f^c_{ab} T^c$. Next, let us choose a representation of the algebra in which $Tr(T^a T^b) \propto \delta_{ab}$ then the non trivial third order charge becomes

$$Q^3_{AdS} = f^a_{bc} \int_0^1 C_a \left(\int_0^1 d\sigma C_b(\sigma) \int_0^\sigma C_c(\sigma')\right), \quad (4.11)$$

where now $C_a$ is the coefficient of $T^a$ in the first order charge, etc.

### 4.2 Penrose limit

The Penrose limit of the Cartan 1-forms were done in [17], where we refer the reader for the details, here we repeat basically their analysis.[9] We define $y^{\mp} = y_{\pm} = (y^9 \pm y^0)/\sqrt{2}$ and then perform the rescaling

$$y^- \to \Omega^2 y^-\,, \qquad y^+ \to y^+\,, \qquad y^{\hat{i}} \to \Omega y^{\hat{i}}\,. \quad (4.12)$$

The Cartan 1-forms should also be rescaled as $L^- \to \Omega^2 L^-$, $L^+ \to L^+$ and $L^{\hat{i}} \to \Omega L^{\hat{i}}$. Finally by performing the following change of coordinates

$$x^{\hat{i}} = \frac{\sin y^+}{y^+} y^{\hat{i}}\,, \quad (4.13)$$

$$x^+ = y^+\,, \quad (4.14)$$

$$x^- = y^- + \frac{y^{\hat{i}} y^{\hat{i}}}{2y^+}\left(1 - \frac{\sin 2y^+}{2y^+}\right), \quad (4.15)$$

we obtain the Cartan 1 forms used in the previous section for the pp-wave as $\Omega \to 0$ (see (3.1)).

$$L^- = dx^- - \frac{1}{2} x^{\hat{i}} x^{\hat{i}} dx^+\,, \qquad L^+ = dx^+\,, \qquad L^{\hat{i}} = dx^{\hat{i}}\,. \quad (4.16)$$

In order to show that $Q^3_{AdS}$ maps into $Q^{NT}_3$ we need also to show that the $AdS_5 \times S_5$ algebra goes to the pp-wave algebra, with the correct structure constants $f^A_{BC}$. This was done in [18], by performing the following rescaling

$$P^+ \to \frac{1}{\Omega^2} P^+\,, \qquad P^{\hat{i}} \to \frac{1}{\Omega} P^{\hat{i}}, \qquad P^{\hat{i}}_* \to \frac{1}{\Omega} P^{\hat{i}}_*\,, \quad (4.17)$$

where $P^a_*$ are the boost generator, and then taking the $\Omega \to 0$ limit. Notice that these rescaling corresponds to the rescaling on the coordinates.

[9] Our conventions interchange + and - with respect to the conventions used in [17].



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So we see that $Q^3_{AdS}$ maps into $Q_3^{NT}$. It is interesting to notice that to construct the charges in $AdS_5 \times S^5$, where we can take the trace of products of operators as invariant form without loosing generality, since the algebra is semi-simple, and then to take their Penrose limit, it is equivalent to consider the charges on the pp-wave but using now the non-degenerate invariant, as done in the previous section.

### 4.3 An explicit check

As an explicit check that the Penrose limit of the charges of $AdS_5 \times S^5$ are the charges on the pp-wave we can consider the following exercise.

Since we have the expression for $Q_3^{NT}$ in terms of the mode expansion of the coordinates $x^I$, we know what is its value when applied to BMN operators:[10]

$$Q_3^{NT}\alpha^\dagger_{n_1}\alpha^\dagger_{n_2}\cdots\alpha^\dagger_{n_L}|0\rangle \sim \left(\frac{1}{n_1}+\frac{1}{n_2}+\cdots+\frac{1}{n_L}\right)\alpha^\dagger_{n_1}\alpha^\dagger_{n_2}\ldots\alpha^\dagger_{n_L}|0\rangle\,, \tag{4.18}$$

where we have taken the classical limit, *i. e.* $\sqrt{1+n^2}\approx 1$.[11] On the other hand, on $AdS_5\times S_5$, the dual of the BMN operators are believed to be rotating strings on an equator of $S^5$ with very large angular momentum. So it is interesting to check the value of $Q^3_{AdS}$ when we plug on it the semiclassical solution corresponding to such rotating string. This was done for the hamiltonian in [20](see also [21]).

In the following we will focus on the $S^5$ part, the analysis for the $AdS_5$ is very much the same. First, let us change coordinates to the one used by [19]. In such coordinates, the metric turns out to be

$$ds^2 = d\phi^2\cos^2\theta + d\theta^2 + \sin^2\theta d\Omega_3^2\,, \tag{4.19}$$

with $d\Omega_3^2$ the metric of a 3-sphere parametrized by angles $\alpha$, $\beta$ and $\gamma$. Performing such change of coordinates and setting $\alpha=\beta=\gamma=0$ for simplicity we can rewrite the coefficients $C_a$ in terms of the new coordinates (As we focus on the $S^5$ only we will suppress the primes)

$$C_{J^{45}} = -C_{J^{54}} = \cos\phi d\theta + \sin\theta\cos\theta\sin\phi d\phi\,, \tag{4.20}$$

$$C_{P^4} = \sin\phi d\theta - \sin\theta\cos\theta\cos\phi d\phi\,, \tag{4.21}$$

$$C_{P^5} = -\cos^2\theta d\phi\,. \tag{4.22}$$

We will consider a string rotating in the equator defined by $\theta=0$, that is, we will consider $\phi = J\tau$ and small oscillations around $\theta=0$. The lagrangian becomes

$$\mathcal{L} = \cos^2\theta(d\phi)^2 + (d\theta)^2 \approx J^2(1+\theta^2) + (d\theta)^2\,. \tag{4.23}$$

---

[10] For future convenience, we use the notation used in [19], where $n>0$ for left movers and $n<0$ for right movers.

[11] More explicitly, reintroducing the dimensionful parameters $\sqrt{m^2+\frac{n^2}{(\alpha' p^+)^2}}\approx m$.

By using the approximation of small perturbation and the explicit form of $\phi$ we obtain

$$
\begin{aligned}
j^A \equiv C_{J^{45}} = -C_{J^{54}} &= \cos J\tau\, d\theta + J \sin J\tau\, \theta d\tau\,, \\
j^B \equiv C_{P^4} &= \sin J\tau\, d\theta - J\cos J\tau\, \theta d\tau\,, \\
j^C \equiv C_{P^5} &= -J d\tau\,.
\end{aligned}
\tag{4.24}
$$

By using the equation of motion

$$
(\partial_\sigma^2 - \partial_\tau^2)\theta - J^2\theta = 0\,, \tag{4.25}
$$

it is easy to see that the quantities $Q^A = \int_0^1 j_0^A$, etc, are conserved. Let us suppose a perturbation of the form[12]

$$
\theta(\tau,\sigma) = \sum_{i=1}^{K} \frac{A_i}{\omega_{n_i}} \left( e^{i(\omega_{n_i}\tau + 2\pi n_i \sigma)} + e^{-i(\omega_{n_i}\tau + 2\pi n_i\sigma)} \right) \tag{4.26}
$$

with $\omega_n = \sqrt{J^2 + 4\pi n^2}$. Then we find

$$
\begin{aligned}
Q^3_{AdS} &= Q^A Q^{[B,C]} + Q^C Q^{[A,B]} + Q^B Q^{[C,A]} \\
&= J\left( \int_0^1 \partial_\tau\theta \int_0^\sigma \theta - \int_0^1 \theta \int_0^\sigma \partial_\tau\theta \right) = \sum_{i=1}^{K} \frac{J^2 A_i^2}{n_i\sqrt{J^2 + 4\pi n_i^2}}\,,
\end{aligned}
\tag{4.27}
$$

that coincides with (4.18) for large J. In fact, one can notice that the form of (4.24) coincides with that of the coefficients $B^I$, $C^I$ and 1, of the pp-wave, from which $Q_3^{NT}$ is built.

Of course this same method can be used to compute this charges for other string states on $AdS_5 \times S^5$ , as done in [20].

## 5. Conclusions

Recently in [4] it was found that the Green-Schwarz superstring on $AdS_5 \times S^5$ possesses an infinite set of non-local classically conserved charges. This would suggest that in some non-trivial cases the world-sheet theory may be exactly solvable. To understand the role of these charges, their gauge dual, etc, seems a very complicated task. As a warm up exersice we propose the study of such charges on the pp-wave limit.

In this paper we show that the closed superstring on pp-waves possesses an infinite tower of non-local classically conserved charges. We then show that they are the Penrose limit of the charges present for the $AdS_5 \times S^5$ background.

In order to construct these charges in closed string theory, one must impose periodic boundary conditions on the world-sheet fields and then take an invariant of the group element appearing in the charge. As a consequence it is not clear whether it is possible to generate all the tower of non-local charges by repeated Poisson Dirac brackets of the first non-local charges (or some finite number of them), as opposed to what happens when one considers the uncompactified sigma model. Indeed, from the first order charges explicitly obtained in this paper it is not possible to obtain more non-local conserved quantities.

[12]The presence of 0-modes will not change the final result.



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On the other hand, when one considers closed string theory on pp-waves, which has a non semi-simple algebra, the naive invariant, i.e. the trace, turns out to be degenerate, and one should look for a non degenerate invariant in order to obtain non-trivial charges. The non degenerate bilinear invariant for the algebra under consideration was found in [13, 14] and we use it in order to compute the first non trivial non-local conserved charge. Remarkably, this charge coincides with the Penrose limit of the first non trivial non-local conserved charge for $AdS_5 \times S^5$, whose algebra is semi-simple and we can use the trace as non-degenerate invariant.

There are many possible further directions to pursue. One could try to complete the analysis for the fermionic sector of the theory, here the simplification obtained by fixing the light cone gauge is more relevant. From the discussion done in this paper, it seems that we cannot generate all the infinite tower of non-local charges from the first non-local charges, at least not as in the case of unbounded sigma model, it could be interesting to show that this is the case in general when one considers closed string theory, or to show how to generate the tower from a finite number of charges. In the case in which the tower cannot be generated one should develop more effective methods that evaluanting the charges order by order. The question about the gauge dual of such charges is interesting. As the AdS/CFT correspondence is more presice in the pp-wave limit (as string theory is exactly solvable in this background) maybe simpler to think about the dual of the charges in this limit.

Even though string theory is exactly solvable on pp-waves, the non-local charges constructed in this paper could provide a clue about the role played by these charges in the full $AdS_5 \times S^5$ background.

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## A. Notation

The commutation relations between the bosonic generators of the $AdS_5 \times S^5$ algebra are

$$\left[P^a, P^b\right] = J^{ab} \qquad \left[P^{a'}, P^{b'}\right] = -J^{a'b'}\,, \tag{A.1}$$

$$\left[P^a, J^{bc}\right] = \eta^{ab}P^c - \eta^{ac}P^b\,, \qquad \left[P^{a'}, J^{b'c'}\right] = \eta^{a'b'}P^{c'} - \eta^{a'c'}P^{b'}\,, \tag{A.2}$$

$$\left[J^{ab}, J^{cd}\right] = \eta^{bc}J^{ad} + 3 \text{ terms} \qquad \left[J^{a'b'}, J^{c'd'}\right] = \eta^{b'c'}J^{a'd'} + 3 \text{ terms}\,, \tag{A.3}$$

with $a = 0, \ldots, 4$, so(4,1) vector indices, in the tangent space of $AdS_5$, $a' = 0, \ldots, 4$ so(5) vector indices in the tangent space of $S^5$, $\eta^{ab} = diag(-++++)$ and $\eta^{a'b'} = diag(+++++)$.

The commutation relations between the bosonic generators of the pp-wave algebra are

$$[P^-, P^I] = -J^{+I}\,,$$

$$[P^I, J^{+J}] = -\delta^{IJ} P^+ \,, \qquad [P^-, J^{+I}] = P^I \,, \tag{A.4}$$

with $I = 1, \ldots, 8$. These generators admit the following field representation

$$P^+ = 1 \,, \qquad P^I = -\int_0^1 d\sigma(\cos\tau\,\partial_\tau x^I + \sin\tau\, x^I) \,, \tag{A.5}$$

$$J^{+I} = \int_0^1 d\sigma(\sin\tau\,\partial_\tau x^I - \cos\tau\, x^I) \,. \tag{A.6}$$

The two dimensional fields $x^I(\tau, \sigma)$ satisfy the following equations of motion and periodicity conditions:

$$(-\partial_\tau^2 + \partial_\sigma^2) x^I - x^I = 0 \tag{A.7}$$

$$x^I(\tau, 0) = x^I(\tau, 1) \,, \qquad \partial_\sigma x^I(\tau, 0) = \partial_\sigma x^I(\tau, 1) \,. \tag{A.8}$$

Such equations admit as solution

$$x^I(\sigma, \tau) = \cos\tau\, x_0^I + \sin\tau\, p_0^I + \mathrm{i} \sum_{n \neq 0} \frac{1}{\omega_n} e^{-\mathrm{i}\omega_n \tau} (e^{\mathrm{i}k_n\sigma} \alpha_n^{1I} + e^{-\mathrm{i}k_n\sigma} \alpha_n^{2I}) \,, \tag{A.9}$$

with the frequencies defined by

$$\omega_n = \sqrt{k_n^2 + 1} \,, \quad n > 0 \,; \qquad \omega_n = -\sqrt{k_n^2 + 1} \,, \quad n < 0 \,, \tag{A.10}$$

$$k_n = 2\pi n, \qquad n = \pm 1, \pm 2, \ldots \tag{A.11}$$

The coordinate $x^-$ can be expressed in terms of $x^I$ by the following constraints

$$\partial_\sigma x^- = -\partial_\tau x^I \partial_\sigma x^I \,, \tag{A.12}$$

$$\partial_\tau x^- = \frac{1}{2}(-\partial_\tau x^I \partial_\tau x^I - \partial_\sigma x^I \partial_\sigma x^I + x^I x^I) \,. \tag{A.13}$$

## References


[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[2] V.M. Braun, S.E. Derkachov and A.N. Manashov, *Integrability of three-particle evolution equations in QCD*, *Phys. Rev. Lett.* **81** (1998) 2020 [hep-ph/9805225];
A.V. Belitsky, *Fine structure of spectrum of twist-three operators in QCD*, *Phys. Lett.* **B 453** (1999) 59 [hep-ph/9902361];
V.M. Braun, S.E. Derkachov, G.P. Korchemsky and A.N. Manashov, *Baryon distribution amplitudes in QCD*, *Nucl. Phys.* **B 553** (1999) 355 [hep-ph/9902375];
A.V. Belitsky, *Renormalization of twist-three operators and integrable lattice models*, *Nucl. Phys.* **B 574** (2000) 407 [hep-ph/9907420]; *Integrability and wkb solution of twist-three evolution equations*, *Nucl. Phys.* **B 558** (1999) 259 [hep-ph/9903512];
S.E. Derkachov, G.P. Korchemsky and A.N. Manashov, *Evolution equations for quark gluon distributions in multi- color QCD and open spin chains*, *Nucl. Phys.* **B 566** (2000) 203 [hep-ph/9909539].




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[3] J.A. Minahan and K. Zarembo, *The Bethe-ansatz for $N = 4$ super Yang-Mills*, *J. High Energy Phys.* **03** (2003) 013 [hep-th/0212208];
A.V. Belitsky, A.S. Gorsky and G.P. Korchemsky, *Gauge/string duality for QCD conformal operators*, *Nucl. Phys.* **B 667** (2003) 3 [hep-th/0304028];
N. Beisert, J.A. Minahan, M. Staudacher and K. Zarembo, *Stringing spins and spinning strings*, *J. High Energy Phys.* **09** (2003) 010 [hep-th/0306139];
N. Beisert, *The complete one-loop dilatation operator of $N = 4$ super Yang-Mills theory*, *Nucl. Phys.* **B 676** (2004) 3 [hep-th/0307015];
N. Beisert and M. Staudacher, *The $N = 4$ SYM integrable super spin chain*, *Nucl. Phys.* **B 670** (2003) 439 [hep-th/0307042];
G. Arutyunov, S. Frolov, J. Russo and A.A. Tseytlin, *Spinning strings in $AdS_5 \times S^5$ and integrable systems*, *Nucl. Phys.* **B 671** (2003) 3 [hep-th/0307191];
A. Gorsky, *Spin chains and gauge/string duality*, hep-th/0308182.

[4] I. Bena, J. Polchinski and R. Roiban, *Hidden symmetries of the $AdS_5 \times S^5$ superstring*, hep-th/0305116.

[5] G. Mandal, N.V. Suryanarayana and S.R. Wadia, *Aspects of semiclassical strings in $AdS_5$*, *Phys. Lett.* **B 543** (2002) 81 [hep-th/0206103].

[6] N. Berkovits, *Super-Poincaré covariant quantization of the superstring*, *J. High Energy Phys.* **04** (2000) 018 [hep-th/0001035].

[7] B.C. Vallilo, *Flat currents in the classical $AdS_5 \times S^5$ pure spinor superstring*, hep-th/0307018.

[8] K. Skenderis and M. Taylor, *Open strings in the plane wave background. I: quantization and symmetries*, *Nucl. Phys.* **B 665** (2003) 3 [hep-th/0211011]; *Open strings in the plane wave background. II: superalgebras and spectra*, *J. High Energy Phys.* **07** (2003) 006 [hep-th/0212184].

[9] R.R. Metsaev and A.A. Tseytlin, *Type-IIB superstring action in $AdS_5 \times S^5$ background*, *Nucl. Phys.* **B 533** (1998) 109 [hep-th/9805028].

[10] R. Kallosh, J. Rahmfeld and A. Rajaraman, *Near horizon superspace*, *J. High Energy Phys.* **09** (1998) 002 [hep-th/9805217].

[11] R. Roiban and W. Siegel, *Superstrings on $AdS_5 \times S^5$ supertwistor space*, *J. High Energy Phys.* **11** (2000) 024 [hep-th/0010104].

[12] R.R. Metsaev, *Type-IIB Green-Schwarz superstring in plane wave Ramond-Ramond background*, *Nucl. Phys.* **B 625** (2002) 70 [hep-th/0112044].

[13] D. Cangemi and R. Jackiw, *Gauge invariant formulations of lineal gravity*, *Phys. Rev. Lett.* **69** (1992) 233 [hep-th/9203056].

[14] C.R. Nappi and E. Witten, *A WZW model based on a nonsemisimple group*, *Phys. Rev. Lett.* **71** (1993) 3751 [hep-th/9310112].

[15] M. Lüscher and K. Pohlmeyer, *Scattering of massless lumps and nonlocal charges in the two-dimensional classical nonlinear sigma model*, *Nucl. Phys.* **B 137** (1978) 46;
E. Brezin, C. Itzykson, J. Zinn-Justin and J.B. Zuber, *Remarks about the existence of nonlocal charges in two- dimensional models*, *Phys. Lett.* **B 82** (1979) 442;
T.L. Curtright and C.K. Zachos, *Nonlocal currents for supersymmetric nonlinear models*, *Phys. Rev.* **D 21** (1980) 411;


JHEP12(2003)033

N.J. MacKay, *On the classical origins of yangian symmetry in integrable field theory*, *Phys. Lett.* **B 281** (1992) 90.

[16] L. Dolan, C.R. Nappi and E. Witten, *A relation between approaches to integrability in superconformal Yang-Mills theory*, *J. High Energy Phys.* **10** (2003) 017 [hep-th/0308089].

[17] M. Hatsuda, K. Kamimura and M. Sakaguchi, *Gauge invariant action for superstring in Ramond-Ramond plane-wave background*, *Nucl. Phys.* **B 644** (2002) 40 [hep-th/0207157].

[18] M. Hatsuda, K. Kamimura and M. Sakaguchi, *From super-$AdS_5 \times S^5$ algebra to super-pp-wave algebra*, *Nucl. Phys.* **B 632** (2002) 114 [hep-th/0202190].

[19] D. Berenstein, J.M. Maldacena and H. Nastase, *Strings in flat space and PP waves from $N = 4$ super Yang-Mills*, *J. High Energy Phys.* **04** (2002) 013 [hep-th/0202021].

[20] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *A semi-classical limit of the gauge/string correspondence*, *Nucl. Phys.* **B 636** (2002) 99 [hep-th/0204051].

[21] J.G. Russo, *Anomalous dimensions in gauge theories from rotating strings in $AdS_5 \times S^5$*, *J. High Energy Phys.* **06** (2002) 038 [hep-th/0205244].

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